

## *Stat 134: Bivariate Normal Review*

*Hank Ibser*

*Dec 6th, 2017*

### *Problem 1: Slices of a Normal Cake*

Let  $X$  and  $Y$  be independent standard normal random variables.

- Find  $P(X > 0, Y > X)$
- Find  $P(X > 0, Y > \sqrt{3}X)$
- Find  $P(X > 0, Y > cX)$  for  $c > 0$

Imagine the relevant volume under the joint density surface, like a slice of a bell-shaped cake.

*Stat 140, HW11, Q6*

### *Problem 2: Like Mother Like Daughter?*

Heights of mothers and daughters in population follow a bivariate normal distribution with correlation 0.5. Both variables have mean 5'4" and standard deviation 2".

- Of daughters with above average height, what percent are shorter than their mothers?
- What percent of daughters are 3 inches taller than their mothers?

*Variation of 6.5.2 in Pitman's Probability*

*Problem 3*

Two random variables  $X$  and  $Y$  are bivariate normal with correlation 0.5. Their marginal distributions are  $X \sim N(1, 1)$  and  $Y \sim N(3, 4)$

- Find  $E(X|Y = 6)$
- Show  $X$  and  $Y - X$  are independent.

*Q7, Stat134, Fall 2015 Final*

What do we know about correlation and independence with bivariate normals?

*Problem 4*

Suppose that  $W$  has normal  $(\mu, \sigma^2)$  distribution. Given that  $W = w$ , suppose that  $Z$  has normal  $(aw + b, \tau^2)$  distribution.

- The joint distribution of  $W$  and  $Z$  is bivariate normal. Find its parameters.
- What is the distribution of  $Z$ ?
- What is the conditional distribution of  $W$  given  $Z = z$ ?

*Ex 6.5.9 in Pitman's Probability*

Hint:  $E(E(Y|X)) = E(Y)$  and  $E(X^2) = \text{Var}(X) + E^2(X)$