Stat 134: Covariance Review

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Fill in the Blanks

a. $Var(X+Y) =$

b. $Cov(X,Y) =$

c. $Cov(\sum_i a_i X_i, \sum_j b_j Y_j) =$

d. $Cov(X,X) =$

e. $Corr(X,Y)$

f. If X and Y are uncorrelated, then $Cov(X,Y) = Cov(X,Y) = 0$. Thus, we write that $\mathbb{E}[XY] =$

g. True or False: If X and Y are independent, X and Y are uncorrelated.

h. True or False: If X and Y are uncorrelated, X and Y are independent.

Problem 2

A fair coin is tossed 300 times. Let H_{100} be the number of heads in the first 100 tosses, and H_{300} the total number of heads in the 300 tosses. Find Corr(H_{100}, H_{300}).
Ex 6.4.10 in Pitman's Probability

Try to use the bilinearity of covariance here.

Problem 3

Suppose there were m married couples, but that d of these 2m people have died. Regard the d deaths as striking the 2m people at random. Let X he the number of surviving couples. Find $\mathbb{E}X$ and $Var(X)$.
Ex 6.4.22 in Pitman's Probability

Problem 4

Suppose n cards numbered $1, 2, ..., n$ are shuffled and k of the cards are dealt. Let S_k be the sum of the numbers on the k cards dealt. Find formulae in terms of n and k for $\mathbb{E}S_k$ and $Var(S_k)$.
Ex 6.4.9 in Pitman's Probability

It might be easiest to use an indicator-esque approach; let $S_k = C_1 + \ldots + C_k$ where C_j is the *value* of the j^{th} card drawn. Note that each of these C_j terms are not actually indicators since their value is not either 0 or 1. In fact, they can take any discrete value from 1 to n.

Problem 5

You have N boxes labeled Box1, Box2, ... , BoxN, and you have k balls. You drop the balls at random into the boxes, independently of each other. For each ball the probability that it will land in a particular box is the same for all boxes, namely $1/n$. Let X_1 be the number of balls in Box1 and X_N be the number of balls in BoxN. Calculate $Corr(X_1, X_N)$.
Ex 6.4.8 in Pitman's Probability

Note that $X_1 + X_2 + \ldots + X_n = k$. What can you conclude about the variance of this sum? How can you use this to find the required correlation?