Stat 134: Section 16

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Problem 1

W, X, Y, Z are independent standard normal random variables. Find without using integrals

a. $P(W + X > Y + Z + 1)$

b. $P(4X + 3Y < Z + W)$

c. $E(4X + 3Y - 2Z^2 - W^2 + 8)$

d. $SD(3Z - 2X + Y + 15)$

Ex 5.3.3 in Pitman's Probability

Problem 2

Suppose the AC Transit bus is scheduled to arrive at my corner at 8: 10 A.M., but its actual arrival time is a normal random variable with mean 8:10 A.M and standard deviation 40 seconds. Suppose I try to arrive at the corner at 8:09, but my arrival time is actually normally distributed with mean 8:09 A.M., and standard deviation 30 seconds.

a. What percentage of the time do I arrive at the corner before the bus is scheduled to arrive?

b. What percentage of the time do I arrive at the corner before the bus does'

c. If I arrive at the stop at 8:09 A.M. and the bus still hasn't come by 8:12 A.M., what is the probability that I have already missed it?

Ex 5.3.7 in Pitman's Probability

Problem 3

Let X, Y be independent standard normal variables. Find:

a. $P(3X + 2Y > 5)$

b. $P(\min(X, Y) < 1)$

c. $P(|\min(X, Y)| < 1)$

d. $P(\min(X, Y) > \max(X, Y) - 1)$

Ex 5.3.6 in Pitman's Probability

Problem 4

Einstein's Model for Brownian Motion: Suppose that the X coordinate of a particle performing Brownian motion has normal distribution with mean 0 and variance σ^2 at time 1. Let X t be the X displacement after time t. Assume the displacement over any time interval has a normal distribution with parameters depending only on the length of the interval, and that displacements over disjoint time intervals are independent.

a. Find the distribution of X_t

b. Let (X_t, Y_t) represent the position at time t of a particle moving in two dimensions. Assume that X_t and Y_t are independent Brownian motions starting at 0 at time $t = 0$. Find the distribution of $R_t = \sqrt{X_t^2 + Y_t^2}$, and give the mean and standard deviation in terms of σ and t.

c. Suppose a particle performing Brownian motion (X_t, Y_t) has an X coordinate after one second which has mean 0 and standard deviation one millimeter (mm). Calculate the probability that the particle is more than 2 mm from the point (0,0) after one second.

Ex 5.3.11 in Pitman's Probability