Stat 134: Section 17

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Problem 1

Suppose I toss three coins. Some of them land heads and some land tails. Those that land tails I toss again. Let X be the number of heads showing after the first tossing, Y the total number showing after the second tossing, including the X heads appearing on the first tossing. So X and Y are random variables such that $0 \leq X \leq Y \leq 3$ no matter how the coins land. Write out distribution tables and sketch histograms for each of the following distributions:

a. the distribution of X;

b. the conditional distribution of Y given $X = x$ for $x = 0, 1, 2,;$

c. the joint distribution of X and Y;

d. the distribution of Y;

e. the conditional distribution of X given $Y = y$ for $y = 0, 1, 2, 3$.

Ex 6.1.1 in Pitman's Probability

Problem 2

Conditioning independent Poisson variables on their sum. Let N_i be independent Poisson variables with parameters λ_i. Think of the N_i as the number of points of a Poisson scatter in disjoint parts of the plane with areas λ_i, where the mean intensity is one point per unit area. What is the conditional joint distribution of $(N_1, \ldots, N_m)$ given $N_1 + \ldots + N_m = n$?
Ex 6.1.6 in Pitman's Probability

Problem 3

Poissonization of the binomial distribution. Let N have Poisson (λ) distribution. Let X be a random variable with the following property: for every n, the conditional distribution of X given $(N = n)$ is binomial (n, p). Show that the unconditional distribution of X is Poisson, and find its parameter.
Ex 6.1.7 in Pitman's Probability