Stat 134: Section 19

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November 20th, 2017

Problem 1

Suppose X has uniform $(0, 1)$ distribution and $P(A \mid X = x) = x^2$.
What is $P(A)$?
Ex 6.3.1 in Pitman's Probability

Problem 2

Conditioning a Poisson process on the number of arrivals in a fixed time. Let T_1 and T_5 be the time of the first and fifth arrivals in a Poisson process with rate λ as in Section 4.2.

a. Find the conditional density of T_1 given that there are 10 arrivals in the time interval (0,1).

b. Find the conditional density of T_5 given that there are 10 arrivals in the time interval (0,1).

c. Recognize the answers to a) and b) as named densities, and find the parameters.

Ex 6.3.10 in Pitman's Probability

Problem 3

Let $S_n = X_1 + ... + X_n$ be the number of successes in a sequence of n independent Bernoulli (p) trials $X_1, X_2, \dots, X_n$ with unknown success probability p. Regard p as the value of a random variable Π.

1. Suppose the prior distribution of Π is beta (r, s) for some $r > 0$ and $s > 0$. Show that the posterior distribution of Π given $S_n = k$ is beta $(r + k, s + n - k)$.

Hint: You only need to show that the posterior distribution is proportional to the beta $(r + k, s + n - k)$ density. Why is this true?

2. Using the fact that the total integral of the beta $(r + k, s + n - k)$ density is 1, find a formula for the unconditional probability $P(S_n = k)$.

Ex 6.3.15 in Pitman's Probability

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