Stat 134: Section 15

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Problem 1

A Geiger counter is recording background radiation at an average rate of one hit per minute. Let T_3 be the time in minutes when the third hit occurs after the counter is switched on. Find $P(2 \leq T_3 \leq 4)$.
Ex 4.2.6 in Pitman's Probability

Can we avoid using integration by parts for this problem?

Problem 2

Let X be a random variable with density $f(x) = 0.5e^{-|x|}$ $(-\infty < x < \infty)$. Find:

a. $P(X < 1)$;

b. $E(X)$ and $SD(X)$;

c. the c.d.f. of X^2.

Ex 4.rev.4 in Pitman's Probability

Recall that the c.d.f. is defined as $F(x) = P(X \leq x)$.

Problem 3

Local calls are coming into a telephone exchange according to a Poisson process with rate λ_{loc} calls per minute. Independently of this, long-distance calls are coming in at a rate of λ_{dis} calls per minute. Write down expressions for probabilities of the following events:

a. exactly 5 local calls and 3 long-distance calls come in a given minute;

b. exactly 50 calls (counting both local and long distance) come in a given three-minute period;

c. starting from a fixed time, the first ten calls to arrive are local.

Ex 4.rev.13 in Pitman's Probability

For c., argue that starting from a fixed time, the number of local calls in the first n calls follows a Binomial$(n, \frac{\lambda_{loc}}{\lambda_{loc}+\lambda_{dis}})$ distribution. For another approach, consider $P(W_1^{loc} + ... + W_{10}^{dis} < W_1^{dis})$, where W_i^{loc} denotes the ith interarrival time for the local calls and W_i^{dis} denotes the ith interarrival time for the long distance calls.

Problem 4

Let Y_1, Y_2, and Y_3 be three points chosen independently and uniformly from $(0, 1)$, and let X be the rightmost (largest) point. Find the c.d.f., density function, and expectation of X.
Ex 4.rev.3 in Pitman's Probability