Stat 134: Section 17

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Problem 1

Let $U_{(1)}, ..., U_{(n)}$ be the values of n independent uniform$(0, 1)$ variables arranged in increasing order. Let $0 \leq x < y \leq 1$. Find simple formulae for:

For a., draw a picture representing the event. For b and c, the answer in part a might be useful. For d, answers in parts a-c might be useful.

a. $P(U_{(1)} > x \text{ and } U_{(n)} < y)$;

b. $P(U_{(1)} > x \text{ and } U_{(n)} > y)$;

c. $P(U_{(1)} < x \text{ and } U_{(n)} < y)$;

d. $P(U_{(1)} < x \text{ and } U_{(n)} > y)$.

Ex 4.6.3a-d in Pitman's Probability

Problem 2

Evaluate the following integrals:

Do not crank them out by calculus; use what you know gamma and beta densities. Recall that the density of $X \sim$ Gamma(r, λ) is $f_X(x) = \frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x}, x > 0$ and 0 otherwise. The density of $Y \sim$ beta(r, s) is $f_Y(y) = \frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)} y^{r-1}(1-y)^{s-1}$ for $0 < y < 1$ and 0 otherwise. The gamma function Γ is defined as $\Gamma(r) = \int_0^\infty x^{r-1} e^{-x} dx$ for $r > 0$. For positive integer n, $\Gamma(n) = (n-1)!$.

a. $\int_0^\infty z^3 e^{-z^2} dz$;

b. $\int_0^\infty x^7 e^{-2x} dx$;

c. $\int_0^{100} x^2 (100 - x)^2 dx$;

Ex 4.rev.13 in Pitman's Probability

Problem 3

C.d.f. of the beta distribution for integer parameters.

Recall that the c.d.f. is defined as $F(x) = P(X \leq x)$. Note that the kth order statistic of n independent uniform$(0,1)$ random variables has beta$(k, n-k+1)$ distribution.

a. Let $X_1, X_2, ..., X_n$ be independent uniform$(0,1)$ random variables, and let $X_{(k)}$ be the kth order statistics of the X's. Find the c.d.f. of $X_{(k)}$ by expressing the event $X_{(k)} \leq x$ in terms of the number of X_i that are $\leq x$.

b. Use a) to show that for positive integers r and s, the c.d.f. of the beta(r,s) distribution is given by

$$\sum_{i=r}^{r+s-1} \binom{r+s-1}{i} x^i (1-x)^{r+s-i-1}, (0 \leq x \leq 1).$$

Ex 4.6.5a-b in Pitman's Probability

Problem 4

A metal rod is l inches long. Measurements on the length of this rod are equal to l plus random error. Assume that the errors are uniformly distributed over the range -0.1 inch and $+0.1$ inch, and independent of each other.

For part b., drawing a picture might help.

a. Find the chance that a measurement is less than $1/100$ of an inch away from l.

b. Find the chance that two measurements are less than $1/100$ of an inch away from each other.

Ex 5.1.2 in Pitman's Probability