Stat 134: Section 18

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Problem 1

Suppose that (X, Y) is uniformly distributed over the region $\{(x, y) : 0 < |y| < x < 1\}$. Find:

Sketch the region. Based on the picture, can you deduce the expectation of Y without any calculation? The picture may also be useful for part c.

a. the joint density of (X, Y);

b. the marginal densities $f_X(x)$ and $f_Y(y)$.

c. Are X and Y independent?

d. Find $E(X)$ and $E(Y)$.

Ex 5.2.1 in Pitman's Probability

Problem 2

A random point (X, Y) in the unit square has joint density $f(x.y) = c(x^2 + 4xy)$ for $0 < x < 1$ and $0 < y < 1$ for some constant c.

a. Evaluate c;

b. Find $P(X \leq a)$, $0 < a < 1$.

c. Find $P(Y \leq b)$, $0 < b < 1$.

Ex 5.2.3 in Pitman's Probability

Problem 3

For random variables X and Y with joint density function

$$f(x,y) = 6e^{-2x-3y}, (x,y > 0)$$

and $f(x,y) = 0$ otherwise, find:

a. $P(X \leq x, Y \leq y)$;

b. $f_X(x)$;

c. $f_Y(y)$;

d. Are X and Y independent? Give a reason for your answer.

Ex 5.2.4 in Pitman's Probability

Problem 4

Minimum and maximum of n independent exponentials. Let $X_1, X_2, ..., X_n$ be independent, each with exponential(λ) distribution. Let $V = \min(X_1, X_2, ..., X_n)$ and $W = \max(X_1, X_2, ..., X_n)$. Find the joint density of V and W.
Ex 5.2.10 in Pitman's Probability

There are two ways to solve the problem: 1. Draw a picture and deduce $P(V \in dv, W \in dw)$. 2. Compute $-\frac{\partial^2}{\partial v \partial w} P(V \geq v, W \leq w)$. Explain why this yields the joint density of (V, W).