Stat 134: Section 19

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Problem 1

W, X, Y and Z are independent standard normal random variables. Find (no integrations are necessary!)

Hint: What are the distributions of $W + X - Y - Z$ and $4X + 3Y - Z - W$? On the other hand, if X and Y are independent, is it true that $SD(X + Y) = SD(X) + SD(Y)$?

a. $P(W + X > Y + Z + 1)$;

b. $P(4X + 3Y < Z + W)$;

c. $E(4X + 3Y - 2Z^2 - W^2 + 8)$;

d. $SD(3Z - 2X + Y + 15)$.

Ex 5.3.3 in Pitman's Probability

Problem 2

Let X and Y be independent standard normal variables. Find:

It may be helpful to draw pictures. For d., what does $\max(X, Y) - \min(X, Y)$ equal to?

a. $P(3X + 2Y > 5)$;

b. $P(\min(X, Y) < 1)$;

c. $P(|\min(X, Y)| < 1)$;

d. $P(\min(X, Y) > \max(X, Y) - 1)$;

Ex 5.3.6 in Pitman's Probability

Problem 3

Suppose heights in a large population are approximately normally distributed with a mean of 5 feet 10 inches and an SD of 2 inches. Suppose a group of 100 people is picked at random from this population.

a. What is the probability that the tallest person in this group is over 6 feet 4 inches tall?

b. What is the probability that the average height of people in the group is over 5 feet 10.5 inches?

c. Suppose instead that the distribution of heights in the population was not normal, but some other distribution with the given mean and SD. To which of the problems a) and b) would the answer still be approximately the same? Explain carefully.

Ex 5.3.9 in Pitman's Probability

Problem 4

A point is picked randomly in space. Its three coordinates X, Y and Z are independent standard normal variables. Let $R = \sqrt{X^2 + Y^2 + Z^2}$ be the distance of the point from the origin. Find:

Recall that if $X_1, ..., X_n$ are i.i.d. standard normal, then $X_1^2 + ... + X_n^2$ is distributed as a χ_n^2 distribution. Note that χ_n^2 is equivalent to $\text{Gamma}(n/2, 1/2)$.

a. the density of R^2;

b. the density of R;

c. $E(R)$;

d. $\text{Var}(R)$.

Ex 5.rev.16 in Pitman's Probability

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