Stat 134: Section 20

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Problem 1

Let X_1 be uniform$(0, 1)$ independent of X_2, that is, uniform$(0, 2)$. Find:

A picture might help.

a. $P(X_1 + X_2 \leq 2)$;

b. the density of $X_1 + X_2$;

c. the c.d.f. of $X_1 + X_2$.

Ex 5.4.1 in Pitman's Probability

Problem 2

A computer job must pass through two queues before it is processed. Suppose the waiting time in the first queue is exponential with rate α, and the waiting time in the second queue is exponential with rate β, independent of the first.

a. Find the density of the total time the job spends waiting in the two queues. Sketch the density in case $\alpha = 1$ and $\beta = 2$.

b. Find the expected total waiting time in terms of α and β.

c. Find the SD of the total waiting time in terms of α and β.

Ex 5.4.3 in Pitman's Probability

Problem 3

A system consists of two components. Suppose each component is subject to failure at constant rate λ, independently of the other, up to when the first component fails. After that moment the remaining component is subject to additional load and to failure at constant rate 2λ.

a. Find the distribution of the time until both components have failed.

b. What are the mean and variance of this distribution?

c. Find the 90th percentile of this distribution.

Ex 5.4.4 in Pitman's Probability

Problem 4

Let X be the number on a die roll, between 1 and 6. Let Y be a random number which is uniformly distributed on $[0, 1]$, independent of X. Let $Z = 10X + 10Y$.

Based on your intuition, what should be the distribution of Z?

a. What is the distribution of Z? Explain.

b. Find $P(29 \leq Z \leq 58)$.

Ex 5.4.5 in Pitman's Probability