Stat 134: Section 21

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Problem 1

Suppose I toss three coins. Some of them land heads and some land tails. Those that land tails I toss again. Let X be the number of heads showing after the first tossing, Y the total number showing after the second tossing, including the X heads appearing on the first tossing. So X and Y are random variables such that $0 \leq X \leq Y \leq 3$ no matter how the coins land. Write out distribution tables and sketch histograms for each of the following distributions:

a. the distribution of X;

b. the conditional distribution of Y given $X = x$ for $x = 0, 1, 2, 3$;

c. the joint distribution of X and Y (no histogram in this case);

d. the distribution of Y;

e. the conditional distribution of X given $Y = y$ for $y = 0, 1, 2, 3$.

f. What is the best guess of the value of X given $Y = y$ for $y = 0, 1, 2, 3$. That is, for each y, choose x depending on y to maximize $P(X = x | Y = y)$.

Ex 6.1.1 in Pitman's Probability

Problem 2

Let X_1 and X_2 be independent Poisson random variables with parameters λ_1 and λ_2.

a. Show that for every $n \geq 1$, the conditional distribution of X_1, given $X_1 + X_2 = n$, is binomial, and the parameters of this binomial distribution.

b. The number of eggs laid by a certain kind of insect follows a Poisson distribution quite closely. It is known that two such insects have laid their eggs in a particular area. If the total number of eggs in the area is 150, what is the chance that the first insect laid at least 90 eggs? (State your assumptions.)

Ex 6.1.5 in Pitman's Probability

Problem 3

Poissonization of the binomial distribution. Let N have Poisson(λ) distribution. Let X be a random variable with the following property: for every n, the conditional distribution of X given $(N = n)$ is binomial(n, p).

a. Show that the unconditional distribution of X is Poisson, and find its parameter.

b. It is known that X-rays produce chromosome breakages in cells. The number of such breakages usually follows a Poisson distribution quite closely, where the parameter depends on the time of exposure, etc. For a particular dosage and time of exposure, the number of breakages follows the Poisson(0.4) distribution. Assume that each breakage heals with probability 0.2, independently of the others. Find the chance that after such an X-ray, there are 4 healed breakages.

Ex 6.1.7 in Pitman's Probability

Problem 4

Suppose you roll a random number of dice. If the number of dice follows the Poisson(λ) distribution, show that the number of sixes is independent of the number of nonsixes.
Ex 6.1.8 in Pitman's Probability

Hint: Let N be the number of dice, X the number of sixes, and Y the number of nonsixes. The previous problem gives you the marginal distributions of X and Y. To show that the joint distribution of X and Y is the product of the marginals, show $P(X = x, Y = y) = P(N = x + y, X = x, Y = y)$ and then use multiplication rule.