Stat 134: Section 22

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Problem 1

Let X_1 and X_2 be the numbers on two independent fair-die rolls. Let X be the minimum and Y the maximum of X_1 and X_2. Calculate:

For discrete X and Y, $E(Y|X=x) = \sum_y yP(Y=y|X=x)$.

a. $E(Y|X=x)$;

b. $E(X|Y=y)$.

Ex 6.2.1 in Pitman's Probability

Problem 2

Repeat Problem 1 with X_1 and X_2 two draws without replacement from $\{1, 2, ..., n\}$.
Ex 6.2.3 in Pitman's Probability

Problem 3

Suppose that N is a counting random variable, with values $\{0, 1, ..., n\}$, and that given $(N = k)$, for $k \geq 1$, there are defined random variables $X_1, ..., X_k$ such that

$$E(X_j | N = k) = \mu, (1 \leq j \leq k).$$

Define a random variable S_N by

$$S_N = \begin{cases} X_1 + X_2 + \cdots + X_k & \text{if } (N = k), 1 \leq k \leq n \\ 0 & \text{if } (N = 0) \end{cases}.$$

Show that $E(S_N) = \mu E(N)$.
Ex 6.2.7 in Pitman's Probability

Problem 4

Hint: Conditioning might help.

A deck of cards is cut into two halves of 26 cards each. As it turns out, the top half contains 3 aces and the bottom half just one ace. The top half is shuffled, then cut into two halves of 13 cards each. One of these packs of 13 cards is shuffled into the bottom half of 26 cards, and from this pack of 39 cards, 5 cards are dealt. What is the expected number of aces among these 5 cards?
Ex 6.2.11 in Pitman's Probability