Stat 134: Section 23

Ani Adhikari

April 24, 2017

Problem 1

Suppose X has uniform $(0,1)$ distribution and $P(A|X = x) = x^2$. What is $P(A)$?
Ex 6.3.1 in Pitman's Probability

Recall that if X is a continuous random variable, $P(B) = \int P(B|X = x) f_X(x) dx$ for any event B.

Problem 2

Suppose (X,Y) has uniform distribution on $R = \{(x,y)|0 \leq y \leq 1 - |x|, -1 \leq x \leq 1\}$. For x between -1 and 1, find:

a. $P(Y \geq 1/2|X = x)$;

b. $P(Y < 1/2|X = x)$;

c. $E(Y|X = x)$;

d. $\text{Var}(Y|X = x)$.

Ex 6.3.5 in Pitman's Probability

Sketch the region R. Note that once $f_Y(y|X = x)$ is found, parts a-d reduce to routine computations.

Problem 3

Suppose that Y and Z are random variables with the following joint density:

Draw a picture and find k first.

$$f(y,z) = \begin{cases} k(z-y) & \text{for } 0 \leq y \leq z \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

for some constant k. Find

a. the marginal distribution of Y;

b. $P(Z < 2/3|Y = 1/2)$.

Ex 6.2.7 in Pitman's Probability

Problem 4

Suppose that a point (X, Y) is chosen uniformly at random from the triangle $\{(x, y) : x \geq 0, y \geq 0, x + y \leq 2\}$. Find a formula for $P(Y \leq y|X = x)$.
Ex 6.3.3 in Pitman's Probability

Prepared by Johnny Hong and Malayandi Palaniappan (Andy)