Stat 134: Section 24

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Problem 1

Let X have uniform distribuiton on $\{-1, 0, 1\}$ and let $Y = X^2$. Are X and Y uncorrelated? Are X and Y independent? Explain carefully.
Ex 6.4.5 in Pitman's Probability

Does uncorrelatedness imply independence in general? Does independence imply uncorrelatedness?

Problem 2

Let X_2 and X_3 be indicators of independent events with probabilities $1/2$ and $1/3$, respectively.

a. Display the joint distribution table of $X_2 + X_3$ and $X_2 - X_3$.

b. Calculate $E(X_2 - X_3)^3$.

c. Are X_2 and X_3 uncorrelated? Prove your answer.

Ex 6.4.7 in Pitman's Probability

Problem 3

Let T_1 and T_3 be the times of the first and third arrivals in a Poisson process with rate λ. Find $Corr(T_1, T_3)$.
Ex 6.4.11 in Pitman's Probability

Recall that $\text{Corr}(X, Y) = \frac{\text{Cov(X, Y)}}{\sqrt{\text{Var}(X)}\sqrt{\text{Var}(Y)}}$. Can you deduce whether $Corr(T_1, T_3)$ is positive or negative without any calculation?

Problem 4

A box contains 5 red halls and 8 blue ones. A random sample of size 3 is drawn without replacement. Let X be the number of red balls and let Y be the number of blue balls selected. Compute: a) $E(X)$; b) $E(Y)$; c) $\text{Var}(X)$; d) $\text{Cov}(X.Y)$.
Ex 6.4.21 in Pitman's Probability

Intuition check: Without any calculation, can you deduce whether $\text{Cov}(X.Y)$ is positive or negative?