

Stat 134: Section 8

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Problem 1

Let X be the number of spades in 7 cards dealt from a well-shuffled deck of 52 cards containing 13 spades. Find $E(X)$.

Ex 3.2.6 in Pitman's Probability

Problem 2

Suppose a fair die is rolled ten times. Find numerical values for the expectations of each of the following random variables:

- a. the sum of the numbers in the ten rolls;
- b. the sum of the largest two numbers in the first three rolls;
- c. the maximum number in the first five rolls;
- d. the number of multiples of three in the ten rolls;
- e. the number of faces which fail to appear in the ten rolls;
- f. the number of different faces that appear in the ten rolls.

Ex 3.2.13 in Pitman's Probability

Hint: For part b., note that the sum of the largest two numbers in the first three rolls is the same as the sum of the first three rolls minus the minimum of the first three rolls.

Problem 3

Aces. A standard deck of 52 cards is shuffled and dealt. Let X_1 be the number of cards appearing before the first ace, X_2 the number of cards between the first and second ace (not counting either ace), X_3 the number between the second and third ace, X_4 the number between the third and fourth ace, and X_5 the number after the last ace. It can be shown that each of these random variables X_i has the same distribution, $i = 1, 2, \dots, 5$, and you can assume this to be true.

- Write down a formula for $P(X_i = k), 0 \leq k \leq 48$.
- Show that $E(X_i) = 9.6$.

Ex 3.2.16ab in Pitman's Probability

For b., do not use your answer to a.

Problem 4

A box contains 3 red balls, 4 blue balls, and 6 green balls. Balls are drawn one-by-one without replacement until all the red balls are drawn. Let D be the number of draws made. Calculate:

- $P(D \leq 9)$;
- $P(D = 9)$;
- $E(D)$.

Ex 3.2.17 in Pitman's Probability

Hint: For c., consider what $13 - D$ represents. The idea in 3.2.16b might be useful.